U. Alabama

Attenuation Constants of Waveguides
With General Cross Sections†

1. Theory

The cutoff frequencies and the field configurations of waveguides with general cross section can be calculated approximately by the point-matching method[1], provided that the method is applicable. With the field configurations of the ideal waveguide (with perfectly conducting guide walls) known, it is expected that the attenuation constant due to the finite conductivity of the guide walls may be estimated numerically.

Conventionally, the attenuation constant is defined as

$$\alpha = P_L / 2 P_T \tag{1}$$

if the guide is made of good conducting material, where P_L is the power loss per unit length. The power transfer P_T is given as[2]

$$P_T = (1/2) \int_S \mathrm{Re} [\overline{E}_t \times \overline{H}_t^* \cdot \overline{z}] \, dS \tag{2}$$

where S is the cross-sectional area of the waveguide, $\overline{z}$ is the unit vector in the propagating direction, and (*) denotes the operation of taking the complex conjugate. The transverse components of the field, E_t and H_t can be calculated from the longitudinal component ψ($\psi = H_z$ for TE modes, $\psi = E_z$ for TM modes), which was obtained by the point-matching method[1]. Substituting the expressions of E_t and H_t in terms of ψ into (2) and after some maniplation the power transfer can be reduced to

$$P_T = G \int_S |\psi|^2 \, dS \tag{3}$$

where $G = (1/2Z_o)(f/f_c)^2 \zeta$ for TM modes

$G = (Z_o/2)(f/f_c)^2 \zeta$ for TE modes

$$\zeta = \sqrt{1 - (f_c/f)^2}$$

and $Z_o = \sqrt{\mu_o / \epsilon_o}$, is the intrinsic impedance of free space. The quantities f_c and f are the cutoff and operating frequencies respectively.

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The power loss per unit length of the guide is conventionally estimated by

$$P_L = (R_s/2) \oint_C |H_{tan}|^2 \, d\ell \tag{4}$$

where $R_s = \sqrt{\omega\mu_o/2\sigma}$, is the surface resistance of the guide wall, σ is the conductivity of the conducting material. The path, C, of the line integral is the contour of the cross section. The integrand in (4) is the square of the magnitude of the magnetic field component tangential to the periphery of the ideal guide walls. Since the normal component of the transverse magnetic field H_t automatically vanishes at the guide surface, it is then possible to express H_{tan} for TM wave modes as follows:

$$|H_{tan}|^2 = |\overline{H}_t (r_c, \theta)|^2 \tag{5}$$

where r_c, a function of θ, describes the cross-sectional contour. For TE wave modes however, the longitudinal component of the magnetic field also contributes to the tangential component. Hence,

$$|H_{tan}|^2 = |\overline{H}_t (r_c, \theta)|^2 + |\psi(r_c, \theta)|^2 \tag{6}$$

The square of the magnitude of the transverse magnetic field, may be written as

$$|\overline{H}_t|^2 = (f/f_c)^2 F(r, \theta) \qquad \text{for TM modes} \tag{7}$$

and

$$|\overline{H}_t|^2 = (f_\zeta/f_c k^2)^2 F(r, \theta) \qquad \text{for TE modes} \tag{8}$$

where

$$F(r, \theta) = \left(\frac{\partial\psi}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial\psi}{\partial\theta}\right)^2$$

Combining (1) through (8) yields the following attenuation constants:

$$\alpha = (R_s/2Z_o \zeta k^2 \int_S |\psi|^2 dS) \oint_C F(r_c, \theta) \, r_c \, d\theta \tag{9}$$

for TM wave modes, and

$$\alpha = (R_s/2Z_o \zeta \int_S |\psi|^2 \, dS)[(\zeta/k)^2 \oint_C F(r_c, \theta) \, r_c \, d\theta + (f_c/f)^2 \oint_C |\psi(r_c, \theta)|^2 \, r_c \, d\theta] \tag{10}$$

for TE wave modes. The integrations in (9) and (10) can be performed numerically.

2. Numerical Example

To demonstrate the validity of the point-matching method for determination of the field distribution, power transfer, and the attenuation constant, it is assumed that an electromagnetic wave is propagating inside a square waveguide in the TE_{10} mode. The guide has a width of 2a and is placed with its center at the origin of a rectangular coordinate system as illustrated in Fig. 1. Since the longitudinal field component H_z is symmetrical with respect to the x-axis for TE_{10}, the sine terms in (1) are omitted. The cutoff wave number calculated by using six points only on the upper half of the guide's cross-sectional contour is 1.5716, compared with the exact value of 1.5708. The expansion coefficients were determined in terms of the coefficient A_1 which is equal to a pre-assigned value of unity. The resulting wave function is therefore expressed in the following form:

$$\psi = H_z = \sum_{n=1}^{3} (-1)^{n+1} J_{2n-1}(kr) \cos(2n-1)\,\theta \tag{11}$$

with three-place accuracy. The disappearance of the even terms in (11) is not surprising because H_z for TE_{10} is antisymmetric with respect to the y-axis. Equation (11) is a good approximation when compared with the exact solution

$$\psi = 0.5 \sin x = \sum_{n=1}^{\infty} (-1)^{n+1} J_{2n-1}(x) \cos(2n-1)\theta \tag{12}$$

since $J_7(kr)/J_1(kr) < 0.001$ for the largest value of r which is $\sqrt{2}\ a$.

The power transported in the waveguide was calculated numerically using (3) and (11), and the result was 0.1256 which is excellent when compared with the exact value of 0.1250.

The attenuation constant is obtained by substituting the approximate wave function (11) into (10) and performing numerical integrations. It is

$$(\alpha_p a Z_o / R_{sc}) = (0.994/2\xi) \sqrt{f/f_c}\ [1 + 2.014 (f_c/f)^2] \tag{13}$$

where $R_{sc} = R_s \sqrt{f_c/f}$, the surface resistance at cutoff frequency, and α_p denotes the attenuation constant of the point matching solution. The exact attenuation constant, α_e, for a square guide of 2a is given by[2]

$$(\alpha_e\ a\ Z_o/R_{sc}) = (1/2\xi)\sqrt{f/f_c}\ [1 + 2(f_c/f)^2] \tag{14}$$

The comparison of these two attenuation constants is shown in Fig. 2 over the frequency range of $(f/f_c) = 1$ through 10. The validity of the attenuation constant calculated by the point matching solutions is then verified.

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References

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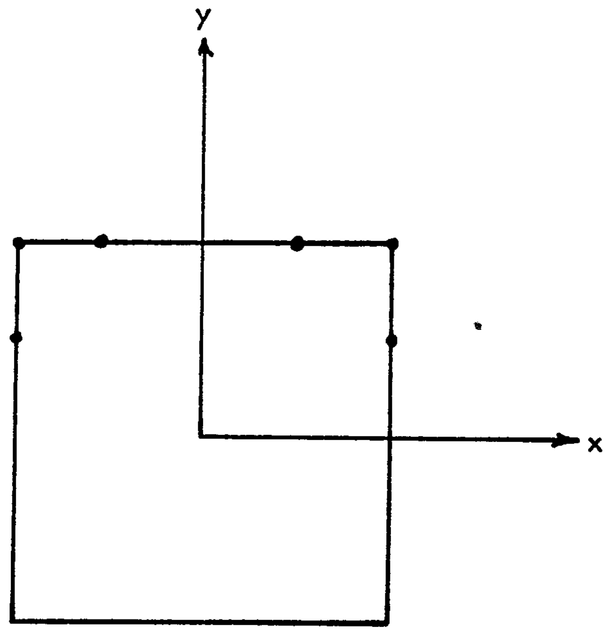

Fig. 1 – The square guide with the six chosen points.

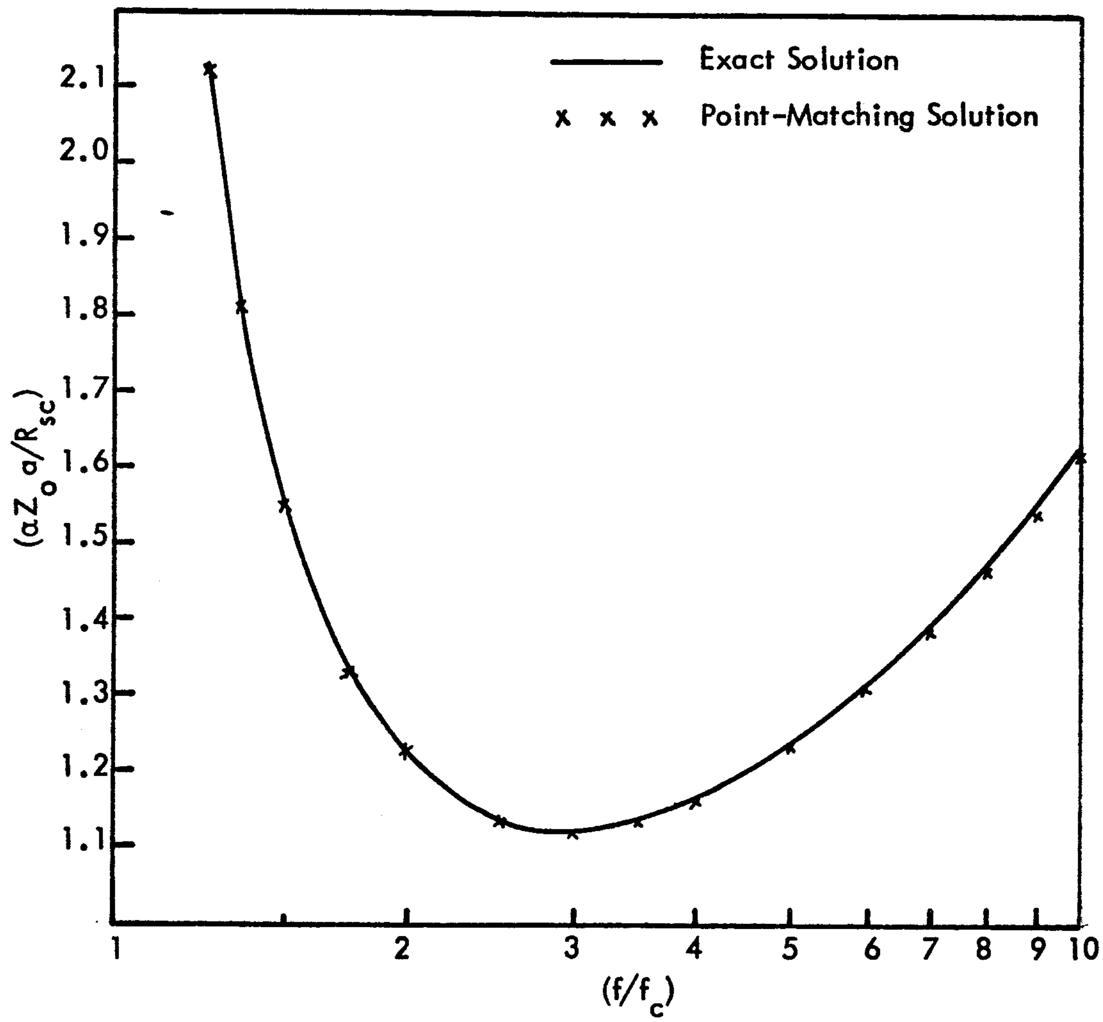

Fig. 2 - The attenuation constants of the square waveguide.